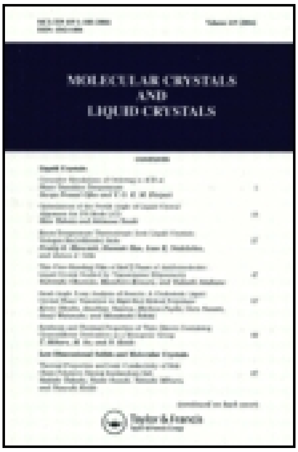

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Optical Defect Modes in Chiral Liquid Crystals at Active Defect Layer

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An analytic approach to the theory of the optical defect modes in chiral liquid crystals (CLC) for the case of an active defect layer is developed. The analytic study is facilitated by the choice of the problem parameters. Namely, an isotropic absorbing or amplifying layer (with the dielectric susceptibility equal to the average dielectric susceptibility of CLC) sandwiched between two CLC layers is studied. The chosen model allows one to get rid off the polarization mixing and to reduce the corresponding equations to the equations for light of diffracting in CLC polarization only. The dispersion equation determining connection of the defect mode frequency with the isotropic layer gain and thickness is obtained. Analytic expressions for the transmission and reflection coefficients of the defect mode structure (CLC-active defect layer-CLC) are presented and analyzed for absorbing and amplifying defect layers. The effect of anomalously strong light absorption at the defect mode frequency for absorbing defect layer is discussed. It is shown that in DFB lasing in a defect structure with an amplifying defect layer an adjusting of the lasing frequency to the DM frequency results in a significant lowering of the lasing threshold and the threshold gain is lowering with increase of defect layer thickness. Numerical estimates of the threshold gain are performed for typical values of the related parameters and analytic expressions for thick CLC layers in the defect structure are presented.

Keywords Anomalous absorption and amplification; localized modes in CLC; low threshold DFB lasing

Introduction

Recently there was an intense activity in the field of mirrorless distributed feedback (DFB) lasing in structures consisting from many layers of chiral liquid crystals (CLC) mainly due to the possibilities to reach a low lasing threshold for the DFB lasing [1–8]. The most part of the related theory is based at the numerical calculations [9] which results are not always interpreted in the frame work of a clear physical picture. Several recent papers [10–13] showed that an analytic theoretical approach to the problem (some times being limited by the introduced approximations) allows to create a clear physical picture of the linear optics and lasing in the mentioned structures. In particular, the physics and role of localized optical modes (edge and defect modes) in the structures under consideration was clearly demonstrated. The most promising results in DFB lasing relates to defect modes (DM)

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[12,13]. The defect modes existing at the structure defect as a localized electromagnetic eigen state with its frequency in the forbidden band gap were investigated initially in the three-dimensionally periodic dielectric structures [14]. The corresponding defect modes in chiral liquid crystals, and more general in spiral media, are very similar to the defect modes in one-dimensional scalar periodic structures. They reveal abnormal reflection and transmission inside the forbidden band gap [1,2] and allow DFB lasing at a low lasing threshold [3]. The qualitative difference with the case of scalar periodic media consists in the polarization properties. The defect mode in chiral liquid crystals is associated with a circular polarization of the electromagnetic field eigen state of the chirality sense coinciding with the one of the chiral liquid crystal helix. There are two main types of defects in chiral liquid crystals studied up to now. One of them is a plane layer of some substance differing from CLC dividing in two parts a perfect cholesteric structure and being perpendicular to the helical axes of the cholesteric structure [1]. Other one is a jump of the cholesteric helix phase at some plane perpendicular to the helical axes (without insertion any substance at the location of this plane) [2]. Recently, a lot of new types of defect layer were studied [15–21], for example, the CLC layer with the pitch differing from the pitch of two layers sandwiching the defect layer [8]. It is evident that there are many versions of the dielectric properties of the defect layer, however, the consideration below will be limited by the first mentioned above types of defect, namely, a layer inserted in a chiral liquid crystals. Main attention will be paid to an active (absorbing or amplifying) defect layer. The reason for that is connected as with the experimental researches of the DFB lasing in CLC where dyes were placed in a defect layer [22] so with a general idea that the unusual properties of DM manifest themselves most clearly just at the middle of defect structure, i.e. at defect layer where intensity of the DM field reaches its maximum. So, we shell assume that there is no absorption in the CLC layers of defect mode structure (DMS) and absorption or amplification of light is happening only in the defect layer. The analytic approach in studying of a DMS with active defect layer is very similar to the previously performed DM studies [12,13], so we shell present below the final results of the present investigation sending the readers for the investigation details to references [12,13].

In the present paper an analytical solution of the defect mode associated with an insertion of an active isotropic layer in the perfect cholesteric structure is presented for light propagating along the helical axes and some limiting cases simplifying the problem are considered.

Defect Mode at Active Defect Layer

To consider the defect mode associated with an insertion of an isotropic layer in the perfect cholesteric structure we have to solve Maxwell equations and a boundary problem for electromagnetic wave propagating along the cholesteric helix for the layered structure depicted at Fig. 1. This investigation was performed in [12,13] under assumption that absorbing or amplifying may be the CLC layers at Fig. 1. So, it is possible to exploit the results of [12,13] for our case of an active isotropic defect layer and nonabsorbing CLC layers introducing only some physically clear changes in the formulas obtained in [12,13]. The assumptions of [12,13] that the average dielectric constant of CLC $?_0$ is coinciding with the dielectric constant of defect layer and external media so the polarization conversion is absent and only light of diffracting circular polarization has to be taken into the consideration is conserved here. As well the main notations of the papers [12,13] are also conserved here.

As is known [9] a lot of information on DM is available from spectral properties of DMS transmission T(d,L) and reflection R(d,L) coefficients.

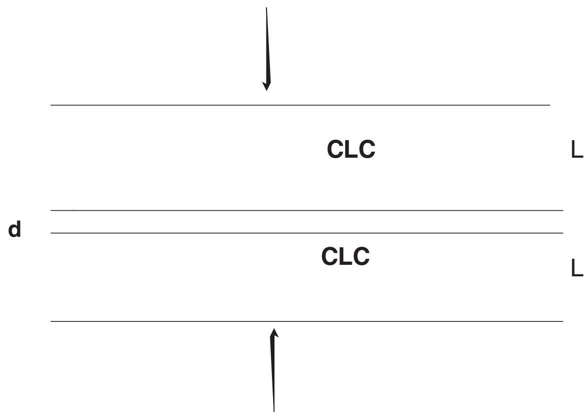


Figure 1. Schematic of the CLC defect mode structure with an isotropic active defect layer of thickness d.

There is an option to obtain formulas determining the optical properties of the structure depicted at Fig. 1 using the expressions for the amplitude transmission T(L) and reflection R(L) coefficient for a single cholesteric layer (see also [23,24]). The transmission $|T(d,L)|^2$ and reflection $|R(d,L)|^2$ intensity coefficients (of light of diffracting circular polarization) for the whole structure may be presented in the following form:

$$|T(d, L)|^2 = |[T_e T_d \exp(ikd(1 + ig))]/[1 - \exp(2ikd(1 + ig))\, R_d R_u]|2, \quad (1)$$

$$|R(d, L)|^2 = |\{R_e + R_u T_e T_u \exp(2ikd(1 + ig))/[1 - \exp(2ikd(1 + ig))R_d R_u]\}|^2, \quad (2)$$

where $R_e(T_e)$, $R_u(T_u)$ and $R_d(T_d)$ are the amplitude reflection (transmission) coefficients of the CLC layer (see Fig. 1) for the light incidence at the outer (top) layer surface, for the light incidence at the inner top CLC layer surface from the inserted defect layer and for the light incidence at the inner bottom CLC layer surface from the inserted defect layer, respectively. It is assumed in the deriving of Eqs. (1,2) that the external beam is incident at the structure (Fig. 1) from the above only. The factor (1+ig) is related to the defect layer only and corresponds to the dielectric constant of the defect layer having the form $\varepsilon_0(1+2ig)$ with a small g being positive for an absorbing defect layer and negative for an amplifying one.

We, for sake of completeness, present here also the expressions for transmission T(L) and reflection R(L) coefficient for a single nonabsorbing cholesteric layer of thickness L for light of diffracting circular polarization [23,24]:

$$R(L) = i\delta \sin qL/\{(q\tau/\kappa^2)\cos qL + i[(\tau/2\kappa)^2 + (q/\kappa)^2 - 1]\sin qL\}$$

$$T(L) = \exp[i\tau L/2](q\tau/\kappa^2)/\{(q\tau/\kappa^2)\cos qL + i[(\tau/2\kappa)^2 + (q/\kappa)^2 - 1]\sin qL\}, \quad (3)$$

where $\kappa = \omega\varepsilon_0{}^{1/2}/c$, $\tau = 4\pi/p$, p is the cholesteric pitch, $\varepsilon_0 = (\varepsilon_{\parallel} + \varepsilon_{\perp})/2$, $\delta = (\varepsilon_{\parallel} - \varepsilon_{\perp})/(\varepsilon_{\parallel} + \varepsilon_{\perp})$ is the dielectric anisotropy, and $\varepsilon_{\parallel}$, $\varepsilon_{\perp}$ are the local principal values of the LC dielectric tensor [23–27],

$$q = \kappa\{1 + (\tau/2\kappa)^2 - [(\tau/\kappa)^2 + \delta^2]^{1/2}\}^{1/2}. \quad (4)$$

The defect mode frequency ω_D is determined by the following dispersion equation:

$$\{\exp(2ikd(1+ig))\sin^2 qL - \exp(-i\tau L)[(\tau q/\kappa^2)\cos qL + i((\tau/2\kappa)^2 + (q/\kappa)^2 - 1)\sin qL]^2/\delta^2]\} = 0. \quad (5)$$

For finite thicknesses of CLC layers L the DM frequency ω_D occurs to be a complex quantity which may be found by a numerical solution of Eq. (5). For a very small values of the parameter g the reflection and transmission spectra of MDS with an active defect layer are similar to the studied in [12,13] spectra (see Fig. 2). In particular, positions of dips in

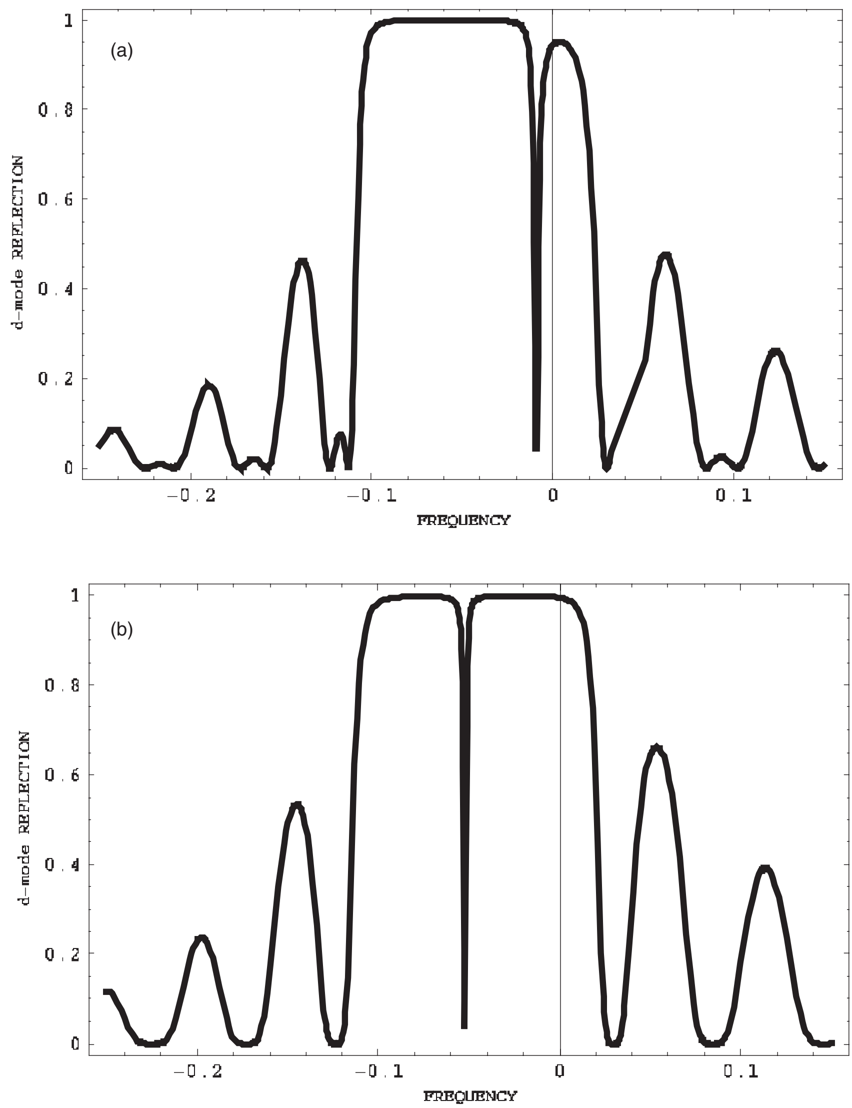
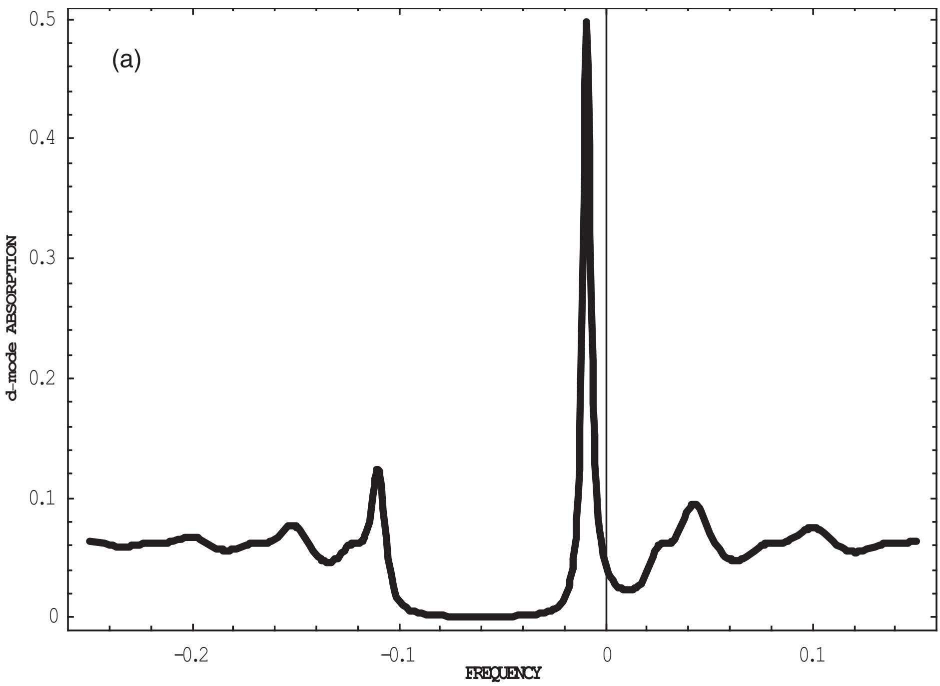


Figure 2. R(d) versus the frequency (Here and at all other figures ? = $\delta[2(\omega\text{-}\omega_B)/(\delta\omega_B) - 1)]$ for a nonabsorbing defect and CLC layers (g = 0) at d/p = 0.1 (a) and d/p = 0.25 (b); $\delta = 0.05$, l = 200, $l = L\tau = 2\pi N$, where N is the director half-turn number at the CLC layer thickness L (at all figures below accepted $\delta = 0.05$ and N = 33).



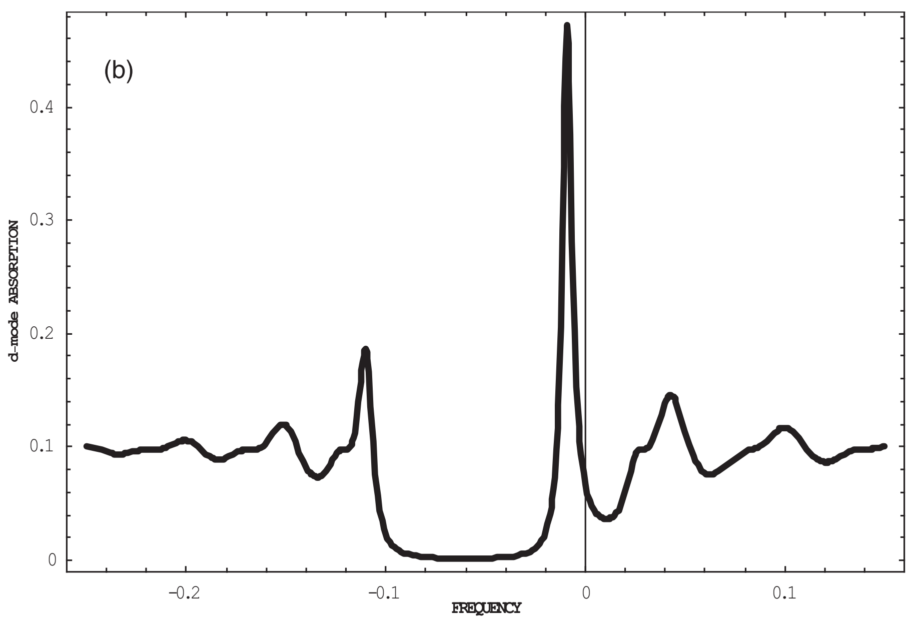


Figure 3. Total absorption versus the frequency for an absorbing defect layer and nonabsorbing CLC layers at g = 0.04978 (a) and g = 0.08 (b) for d/p = 0.1; at g = 0.00008891 (c) and at g = 0.0008891 (d) for d/p = 22.25.

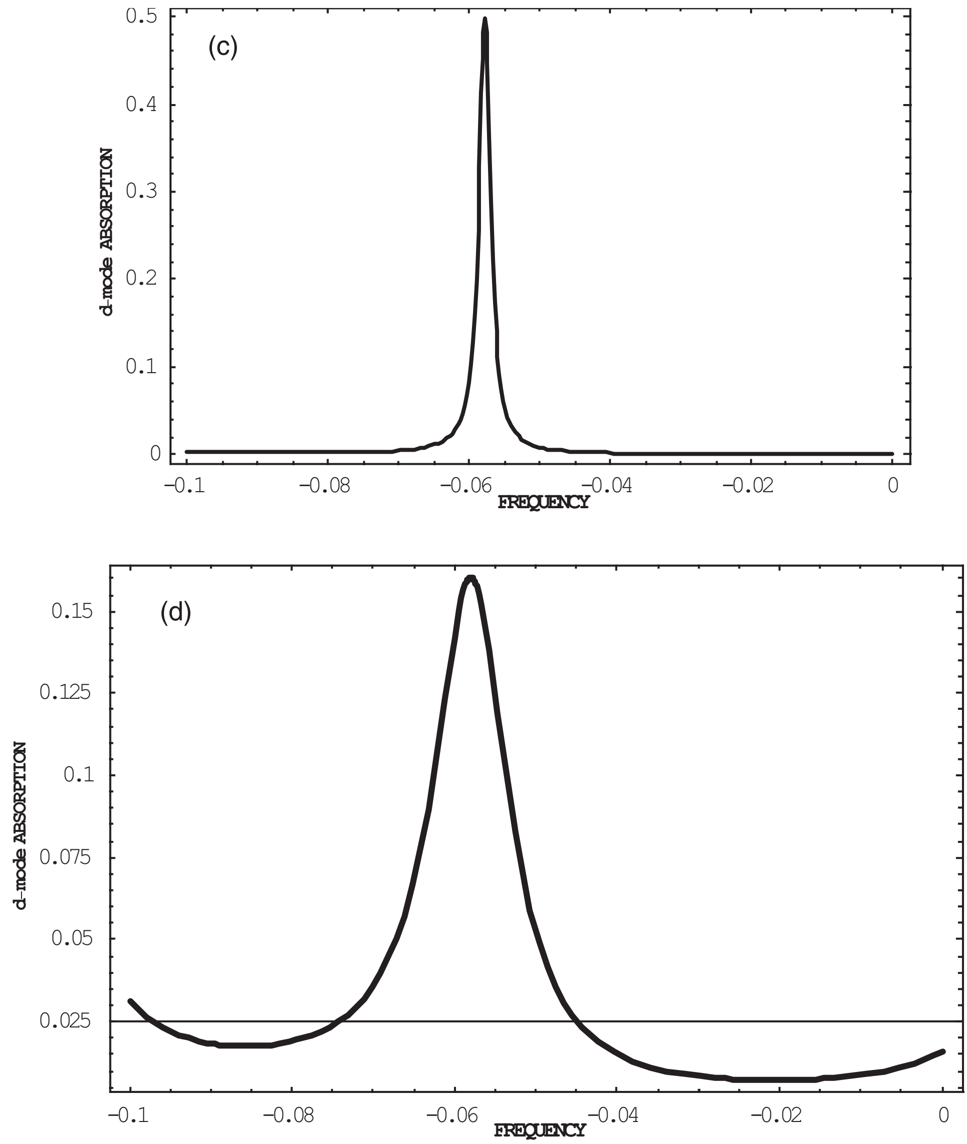


Figure 3. (Continued)

reflection and spikes in transmission inside the stop-band just correspond to Re[ω_D] and this observation is very useful for numerical solution of the dispersion equation. What is concerned of the DM life-time it reduces for absorbing defect layers compared to the case of nonabsorbing defect layer [12,13].

Absorbing Defect Layer

As in the case of investigated DMs with absorbing CLC layers [12,13] in DMS with absorbing defect layer the effect of anomalously strong absorption takes place. The effect reveals itself at the DM frequency and reaches its maximum (maximum of 1-|T(d,L)|^2-|R(d,L)|^2) for definite value of g which may be found using the expressions (1,2) for |T(d,L)|^2 and |R(d,L)|^2. Figure 3 demonstrate existence of the anomalously strong absorption effect. As follows from Fig. 3 the maximum values of the anomalous absorption [23,28] (maximum of 1-|T(d,L)|^2-|R(d,L)|^2) at two differing values are reached for g = 0.04978 g = 0.0008891 (just found in the next section with opposite sign of g as approximate values of the lasing threshold gain for the same DMSs).

In the case of thick CLC layers (|q|L $\gg$ 1) in the DMS the g value ensuring absorption maximum may be found analytically:

$$g_t = (L/d)|[2\kappa^2/(q^2\tau L)]\exp[-2|q|L]\{1 + \{1/(2[(\tau/\kappa)^2 + \delta^2]^{1/2})$$
$$- (\tau/2\kappa)^2\}/(1 - [(\tau/\kappa)^2 + \delta^2]^{1/2} + (\tau/2\kappa)^2)\} - 1|. \quad (6)$$

For the defect mode frequency ω_D in the middle of stop-band the maximal absorption corresponds to

$$g_t = (2/3\pi)(p/d)\ \exp[-2\pi\delta(L/p)]. \quad (7)$$

As the calculations and the formulas (6,7) show the gain (g) corresponding to the maximal absorption is approximately inversely proportional to the defect layer thickness d.

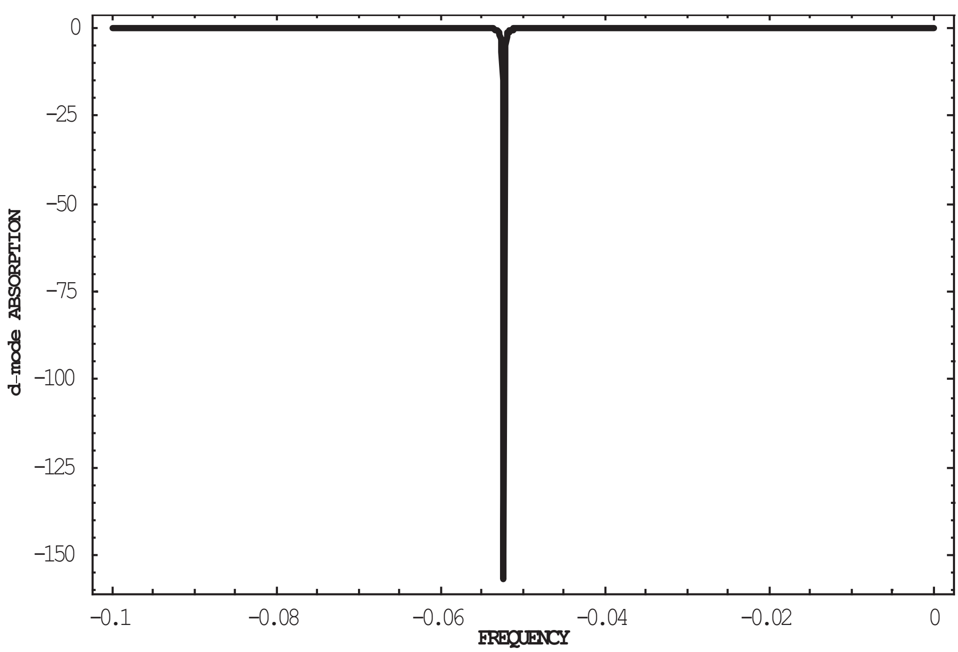


Figure 4. Total absorption 1-|T(d,L)|^2-|R(d,L)|^2 versus the frequency for amplifying defect layer and nonabsorbing CLC layers at g = −0.0065957 for d/p = 0.25.

Amlifying Defect Layer

In the case of DMS with amplifying defect layer ($g < 0$) at some value of $|g|$ divergences of reflection and transmission coefficients occur. The corresponding values of g are the gain lasing thresholds. Their values may be found numerically using the expressions (1,2) for $|T(d,L)|^2$ and $|R(d,L)|^2$ or found approximately by plotting $|T(d,L)|^2$ and $|R(d,L)|^2$ at varying g. The second options is illustrated by Figs 4–6 where "almost divergent" values of $|T(d,L)|^2$, $|R(d,L)|^2$ or absorption ($1-|T(d,L)|^2-|R(d,L)|^2$) are shown. The used values of g

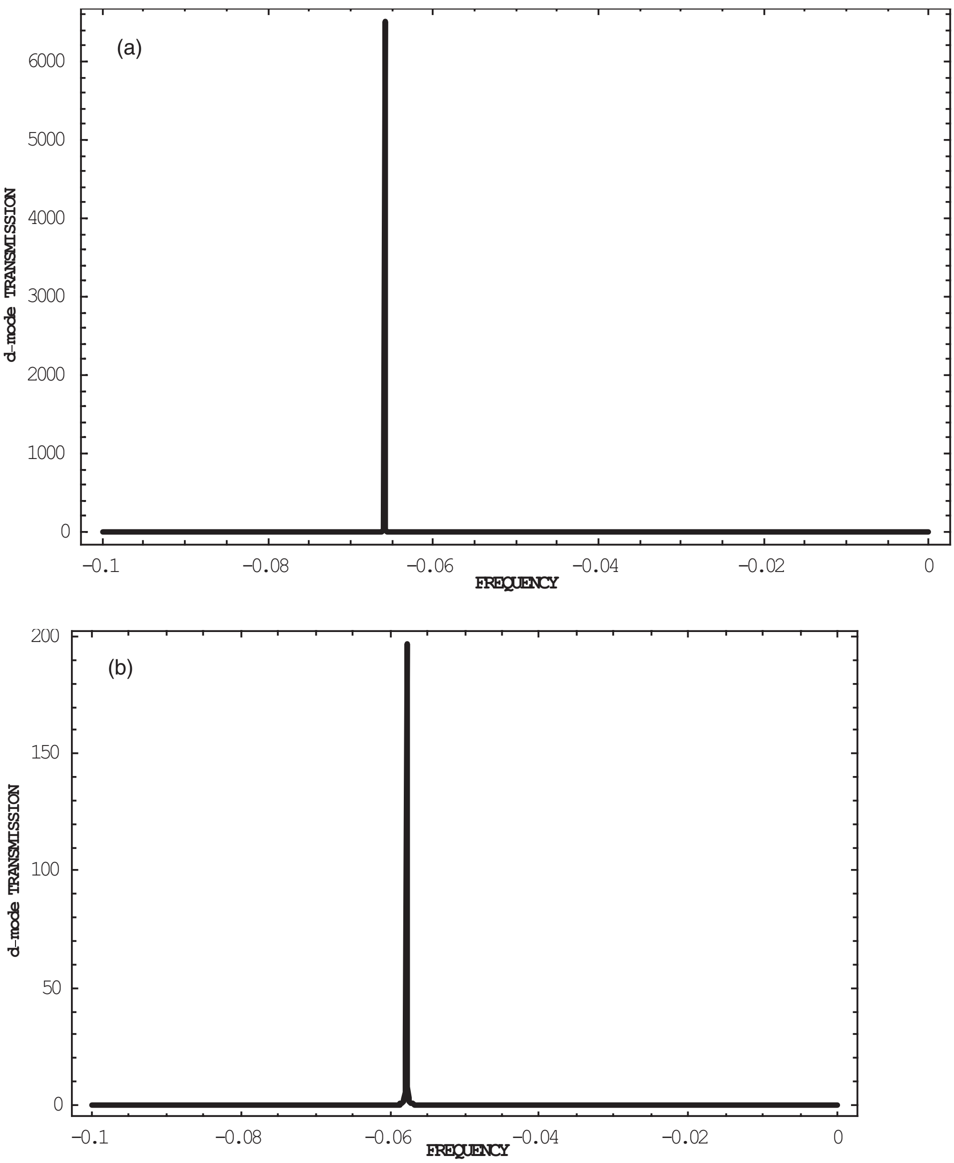


Figure 5. $|T(d)|^2$ versus the frequency for amplifying defect layer and nonabsorbing CLC layers at $g = -0.001000$ for $d/p = 2.25$ (a); at $g = -0.00008891$ for $d/p = 22.25$ (b).

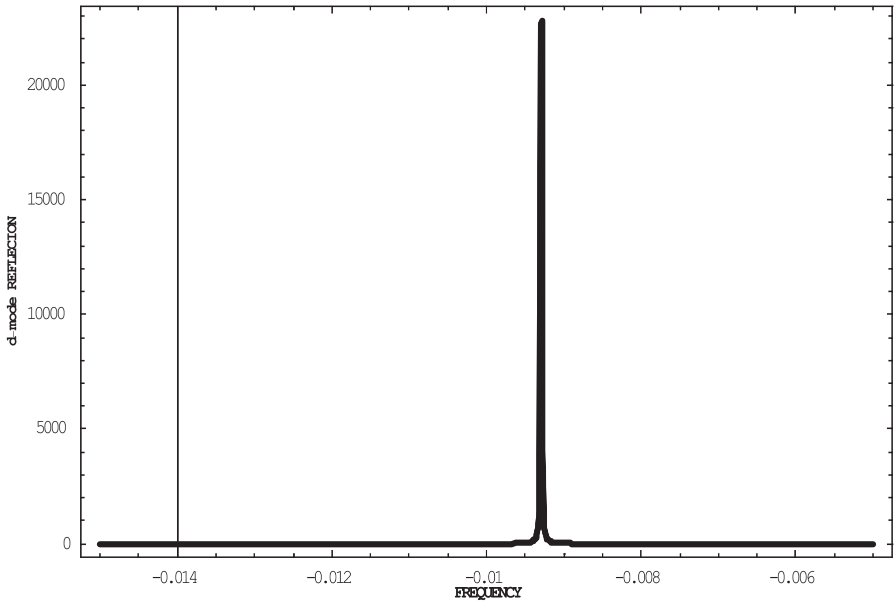


Figure 6. $|R(d)|^2$ versus the frequency for amplifying defect layer and nonabsorbing CLC layers at $g = -0.04978$ for $d/p = 0.01$.

at Figs 4–6 are close to the threshold ones ensuring divergence of $|T(d,L)|^2$ and $|R(d,L)|^2$. The calculation results show that the minimal threshold $|g|$ corresponds to location of ω_D just in the middle of the stop-band and $|g|$ is almost inversely proportional to the defect layer thickness. Really, the Figs 4 and 5 correspond to location of the defect mode frequency ω_D close to the middle point of the stop band and demonstrate decrease of the lasing threshold gain with increase of the defect layer thickness. Figure 6 corresponds to location of the defect mode frequency ω_D close to the stop band edge and demonstrates increase of the lasing threshold gain with approaching the defect mode frequency ω_D to the stop band edge.

The analytic approach for thick CLC layers ($|q|L \gg 1$) results in the similar predictions, namely, the threshold value of gain is given by (6) with negative sign of the right hand side of the expression. For the thick CLC layers for ω_D in the middle of the stop-band the threshold gain is given by the expression:

$$g_t = -(2/3\pi)(p/d)\exp[-2\pi\delta(L/p)]. \tag{8}$$

So, as the formula (8) shows the thinner defect layer is the higher is threshold gain g.

The same result, as was mentioned above, relates also to the absorption enhancement (formula (6,7)). The thinner defect layer is the higher is g value ensuring maximal absorption.

Conclusion

The performed in the previous sections analytical description of the defect modes at an active defect layer neglecting the polarization mixing at the boundaries of CLC in the structure under consideration allows one to reveal clear physical picture of these modes which is applicable to the defect modes in general. For example, more low lasing threshold and more strong absorption (under the conditions of anomalously strong absorption effect) at the defect mode frequency at the middle of stop-band compared to the defect mode frequency close to the stop-band edge are the features of any periodic media. For a special choice of the parameters in the experiment the obtained formulas may be directly applied to the experiment. However, in the general case one has take into account a mutual transformation at the boundaries of the two circular polarizations of opposite sense. For example, the observed in the experiment [3] circular polarization sense of the emitted from the defect structure wave above the lasing threshold may be opposite to the polarization sense responsible for the defect mode existence. Evident explanation of the "lasing" at the opposite (nondiffracting) circular polarization is in the following. Due to the polarization conversion of the generated wave into a wave of opposite circular polarization the converted wave of a nondiffracting polarization freely escapes from the structure. This polarization conversion phenomenon adds also a contribution to the frequency width of the defect mode.

In the conclusion should be stated that the results obtained here for the defect modes with an active defect layer clarify the physics of these modes. An important result relating to the DFB lasing at DMS with active defect layer may be formulated as the following. The lasing threshold gain in defect layer decreases with the layer thickness increase being almost inversely proportional to its thickness. The similar result relates to the effect of anomalously strong absorption phenomenon where the value of g in the defect layer ensuring a maximal absorption is almost inversely proportional to the defect layer thickness. Note, that the revealed here decreasing of the lasing threshold gain with increasing of an amplifying defect layer thickness may not be regarded directly as the corresponding reduction in the lasing energy threshold of a pumping wave pulse. The situation depends on the specific of pumping arrangement. This question demands more specific separate consideration. For example, if one assumes that the pumping is arranged in a such way that the product of the gain g and the defect layer thickness d is proportional to the pumping pulse energy the threshold pumping pulse energy occurs to be almost independent on the defect layer thickness because the found above almost inverse proportionality of the threshold gain to the defect layer thickness.

Note that the obtained above results are qualitatively applicable to the corresponding localized electromagnetic modes in any periodic media and may be regarded as a useful guide in the studies of the localized modes with an active defect layer in general.

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